To: Professor Younis

From: Mason Averill

Subject: Design Problem 1

Date: 29 October 2020

Purpose:

The purpose of this memo is to communicate the methodology and results of Design Problem 1 for ME-550: Advanced Stress Analysis, completed October 29th 2020.

Purpose and Scope of Assignment:

The purpose of this assignment was to determine the maximum value of ‘P’, see Figure 1, before the tube would fail according to multiple different failure theories. In addition, this process was to be repeated for numerous values of ‘d’, see Figure 2.

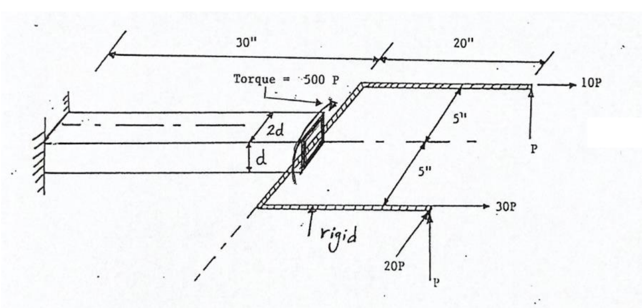

Figure 1: Loading Configuration

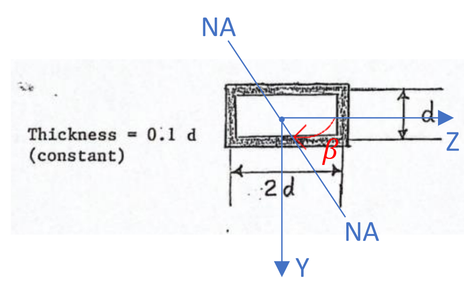

Figure 2: Cross-Section Properties

Method of Analysis:

The first step in the analysis of this problem involved determining the location of the critical cross section—that is, determining the position along the length of the tube that would be subjected to the largest magnitude of stresses. In order to determine this location, the stress situation was analyzed using the following procedure:

1. Any axial stress due to ‘P’ would be uniform throughout the tube, so this does not aid in determining where the critical cross-section will be.
2. Any shear stress due to torsion would be uniform (comparing one cross-section to another) throughout the length of the tube, so this does not aid in determining where the critical cross-section will be.
3. The maximum bending stress would occur at the location of the maximum internal moment in the tube. By analyzing Figure 1, it is evident that this would occur at the cross-section where the tube is fixed to the wall.

Next, with the location of the critical cross-section determined, it was now necessary to determine the internal loads at this section. By using static equilibrium equations, the magnitude of the internal loads were found to be as follows:

- $P_{axial} = 40P$
- $Torque = 500P$
- $M_y = -1100P$
- $M_z = 100P$

Next, with the second area moments of inertia about the Y and Z axes computed in terms of ‘d’, the orientation of the neutral axis was determined using Equation 1.

$$\beta = tan^{-1}\left(-\frac{M_y * I_{zz}}{M_z * I_{yy}}\right) = \cong 75.49^{\circ} \qquad \text{Equation 1}$$

With:

- $\beta = Angle\ from\ positive\ Z\ towards\ positive\ Y\ (Degrees)$
- $M_y = Bending\ Moment\ about\ the\ Y - axis\ (lbf * in)$
- $M_z = Bending\ Moment\ about\ the\ Z - axis\ (lbf * in)$
- $I_{yy} = Second\ Area\ Moment\ of\ Inertia\ about\ the\ Y - axis\ (in^4)$
- $I_{zz} = Second\ Area\ Moment\ of\ Inertia\ about\ the\ Z - axis\ (in^4)$

Note that a direct result for the orientation for the neutral axis was attainable without knowing the values of ‘P’ and ‘d’ because both ‘P’ and ‘d’ were able to be factored out in the numerator and denominator and cancel eachother out.

Next, it was necessary to determine the points on the cross section where the maximum tensile and compressive stress would occur. By reviewing the signs associated with the bending moments, it is evident that the maximum tensile stress would occur at point 'A' in Figure 3 and the maximum compressive stress would occur at point 'B' in Figure 3.

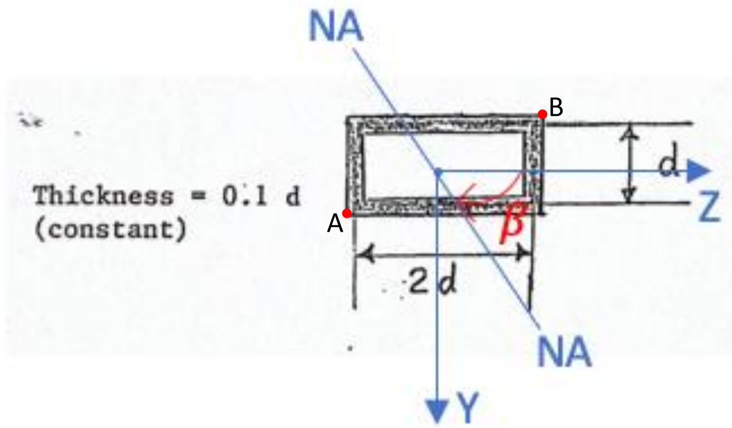

Figure 3: Locations of Maximum Tensile and Compressive Stresses

Next, the stresses acting on the critical cross-section were computed in terms of 'd' and 'P' at both points A and B. The results are as follows:

For Point A:

- $\sigma_{axial_x} = \frac{P_{axial}}{Area_{cross-section}} = \frac{40P}{(2d+0.1d)*(d+0.1d)-(2d-0.1d)*(d-0.1d)}$
- $\tau_{Torsion} = \frac{Torque}{2*Area_{mean}*thickness} = \frac{500P}{2*(2d*d)*(0.1d)}$
- $\sigma_{bending_x} = \frac{M_z*y_A}{I_{zz}} + \frac{M_y*z_A}{I_{yy}} = \frac{100P*(\frac{d}{2}+\frac{0.1d}{2})}{I_{zz}} + \frac{-1100P*-(d+\frac{0.1d}{2})}{I_{yy}}$

For Point B:

- $\sigma_{axial_x} = \frac{P_{axial}}{Area_{cross-section}} = \frac{40P}{(2d+0.1d)*(d+0.1d)-(2d-0.1d)*(d-0.1d)}$
- $\tau_{Torsion} = \frac{Torque}{2*Area_{mean}*thickness} = \frac{500P}{2*(2d*d)*(0.1d)}$
- $\sigma_{bending_x} = \frac{M_z*y_A}{I_{zz}} + \frac{M_y*z_A}{I_{yy}} = \frac{100P*-(\frac{d}{2}+\frac{0.1d}{2})}{I_{zz}} + \frac{-1100P*(d+\frac{0.1d}{2})}{I_{yy}}$

Next, the principal stresses were computed at both points 'A' and 'B' using Equation 2.

$$\sigma_{P_1,P_2} = \frac{\sigma_{bending_x} + \sigma_{axial_x}}{2} \pm \sqrt{(\frac{\sigma_{bending_x} + \sigma_{axial_x}}{2})^2 + \tau_{Torsion}{}^2} \qquad \text{Equation 2}$$

With:

- $\sigma_{P_1,P_2} = Principal\ Stress\ 1\ and\ 2\ (Psi)$

By reviewing Equation 2, it is evident that one principal stress will be positive and one principal stress will be negative since $\tau_{Torsion} \neq 0$. Note that the principal stresses are still in terms of ‘P’ and ‘d’.

Finally, a value of ‘d’ was selected and ‘P’ was solved for based on the 5 following failure theories:

1. The Maximum Normal Stress Theory
   - i $\sigma_{max-allowable} = \sigma_{yield\ strength}$
   - ii $\sigma_{max} = \sigma_{P_1} = \sigma_{max-allowable}$
2. The Maximum Shear Stress Theory
   - i $\tau_{max-allowable} = \frac{\sigma_{yield\ strength}}{2}$
   - ii $\tau_{max} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2} = \tau_{max-allowable}$
3. The Maximum Normal Strain Theory
   - i $\epsilon_{max-allowable} = \frac{1}{E} * \sigma_{yield\ strength}$
   - ii $\epsilon_{max} = \frac{1}{E} * (\sigma_{P_1} - v * \sigma_{P_2}) = \epsilon_{max-allowable}$
4. The Maximum Distortion Energy Theory
   - i $U_{d_{allowable}} = \frac{1+v}{6*E} * 2 * (\sigma_{yield\ strength})^2$
   - ii $U_{d_{max}} = \frac{1+v}{6*E} * [(\sigma_{P_1} - \sigma_{P_2})^2 + \sigma_{P_1}{}^2 + \sigma_{P_2}{}^2] = U_{d_{allowable}}$
5. The Octahedral Shear Stress Theory
   - i $\tau_{max-allowable\ oct} = \frac{1}{3} * \sqrt{2} * \sigma_{yield\ strength}$
   - ii $\tau_{max\ oct} = \frac{1}{3} * \sqrt{[(\sigma_{P_1} - \sigma_{P_2})^2 + \sigma_{P_1}{}^2 + \sigma_{P_2}{}^2]} = \tau_{max-allowable\ oct}$

With v being Poisson’s ratio, E being Young’s Modulus, and $\sigma_{yield\ strength}$ being the yield strength of the material comprising the tube.

This process was repeated using MATLAB for $0.1 \leq d \leq 20$ at a step size of 0.4. Note that the MATLAB script implemented allows for easy manipulation of the range for which values of d should be considered as well as the step size. This step size was selected because it allows for easier viewability of the Figures generated when exported to Microsoft Word.

<u>Results:</u>

Figure 4 shows the results of the maximum allowable magnitude of 'P' vs 'd' at point 'A' when the yield strength is 36,300 Psi (in compression and tension), Poisson's ratio is 0.26, and Young's Modulus is 29MPsi (note that a value of Young's Modulus is not required as it cancels out when solving for 'P').

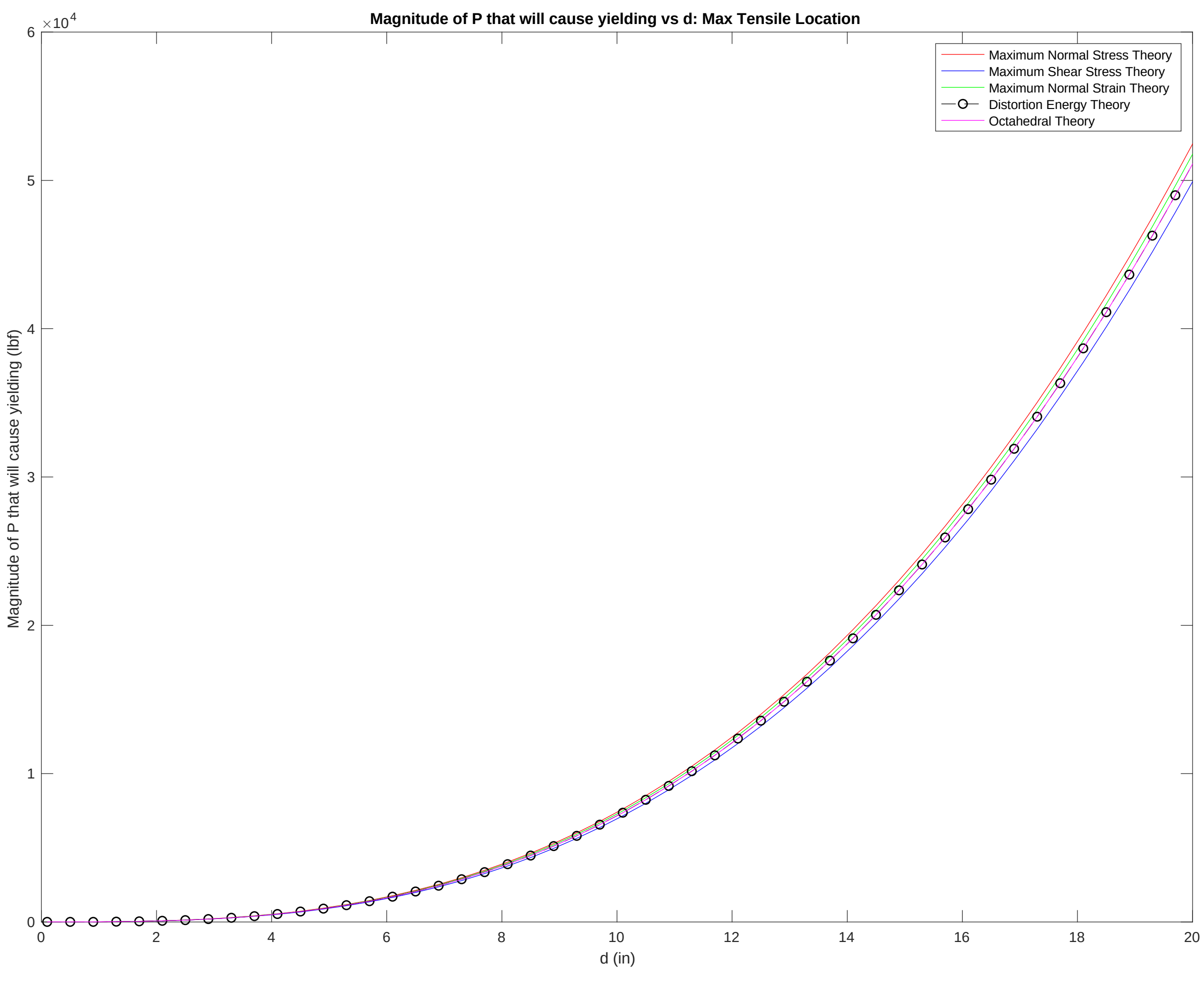

Figure 4: Maximum Allowable P vs d at Point A

Figure 5 shows the results of the maximum allowable magnitude of ‘P’ vs ‘d’ at point ‘B’ using the same material properties. Note that the Maximum Normal Stress Theory and Maximum Normal Strain Theory is not applicable at this point and thus will not be included in the plot.

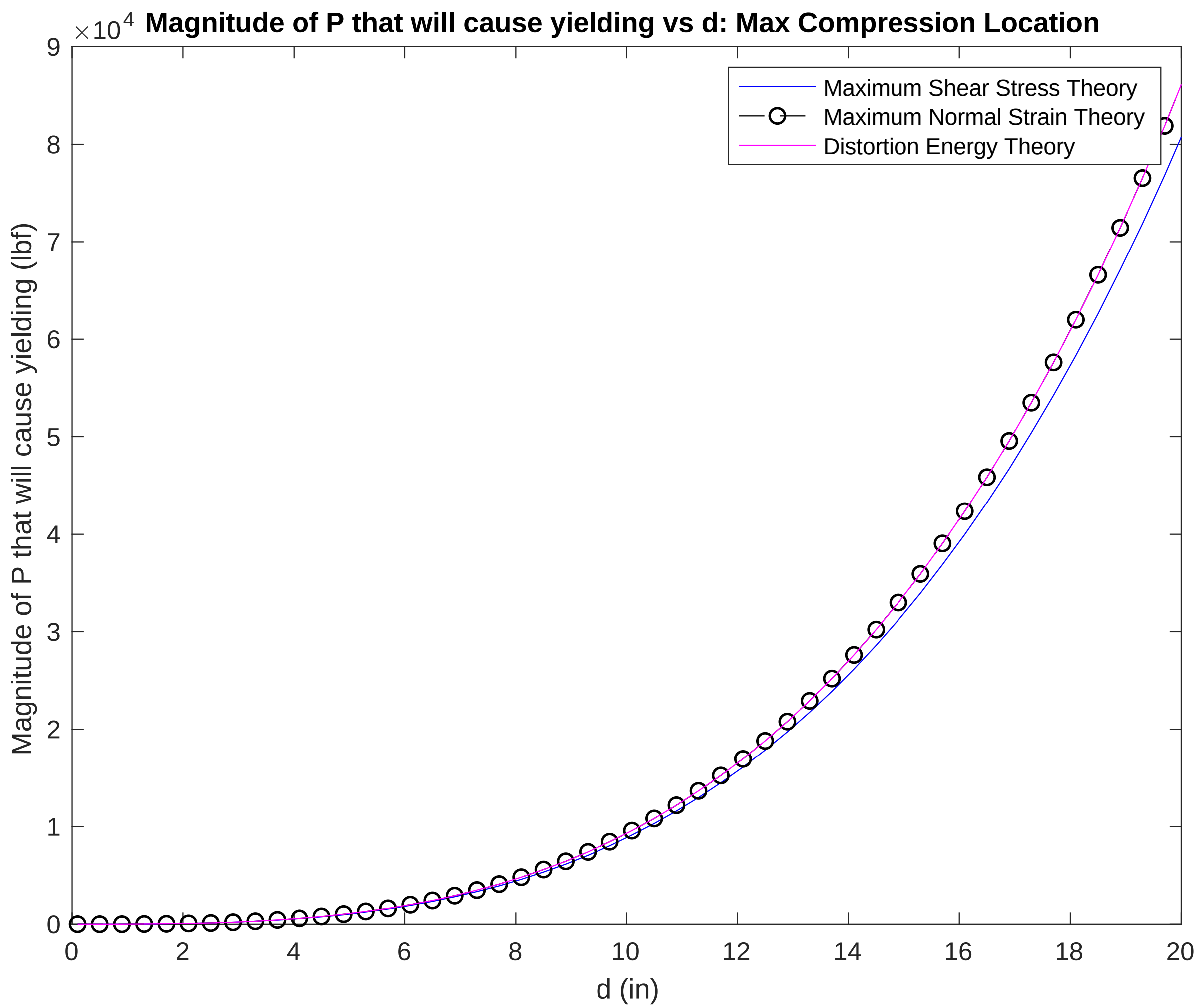

Figure 5: Maximum Allowable P vs d at Point B

By reviewing Figures 4 and 5 it is evident that the tube will fail in tension before it does in compression, as would be expected by reviewing the stresses acting on the tube.

Figure 6 shows a zoomed in view of Figure 4 at d equals 20in, with a step size of 0.1 in order to increase the resolution.

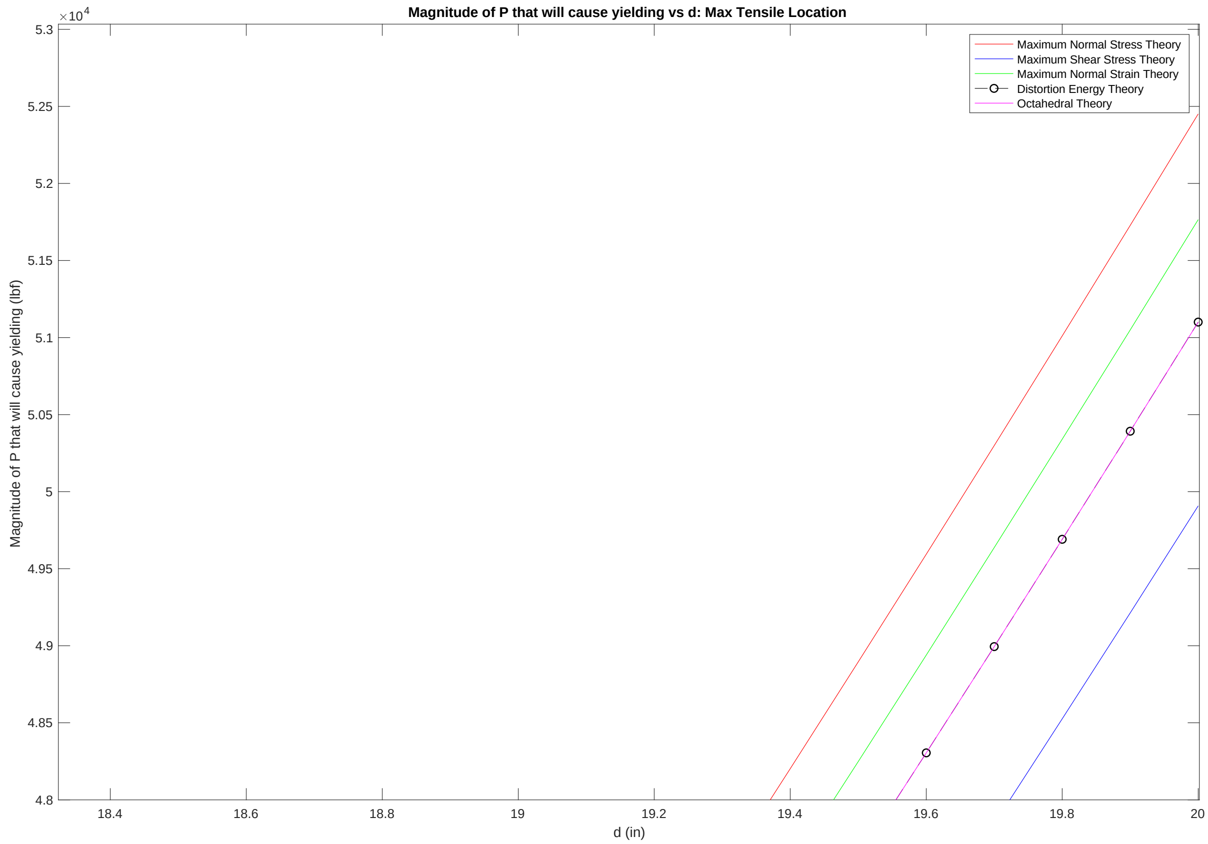

Figure 6: Maximum Allowable P vs d at Point A-Zoomed In

By reviewing Figures 4-6, it is evident that The Maximum Distortion Energy Theory and The Maximum Octahedral Shear Stress Theory predict failure for equal values of 'P'. Figure 6 helps to better visualize the differences between the maximum value of P that can be applied before failure occurs according to each of the different theories. Note that the trend demonstrated in Figure 6 is true for all values of d, the curves do not intersect at any point and remain in the same order as shown. At d equals 20 in, the results obtained are as follows for point 'A':

1. The Maximum Normal Stress Theory: P_{max} =52451.448 lbf
2. The Maximum Shear Stress Theory: $P_{max} = 49907.504$ lbf
3. The Maximum Normal Strain Theory: P_{max} = 51765.400 lbf
4. The Maximum Distortion Energy Theory: P_{max} = 51100.540 lbf
5. The Octahedral Shear Stress Theory: P_{max} = 51100.540 lbf

This Maximum Shear Stress Theory predicts the lowest value of 'P' before failure and the Maximum Normal Stress Theory predicts the highest value of 'P' before failure. The difference between these two values is 2543.940 lbf. Compared to the average value of all the failure theories, this is about a 5% deviation overall, or $\pm$ 2.5%.

At d equals 20 in, the results obtained are as follows for point 'B':

1. The Maximum Shear Stress Theory: $P_{max} = 80710.215$ lbf
2. The Maximum Distortion Energy Theory: $P_{max} =$ 86071.300 lbf
3. The Octahedral Shear Stress Theory: $P_{max} =$ 86071.300 lbf

Again, the Maximum Shear Stress Theory predicts the lowest value of 'P' before failure occurs. The Maximum Distortion Energy Theorem and The Octahedral Shear Stress Theory predict the same maximum value for 'P' before failure occurs. The difference between these two values is 5361.090 lbf. Compared to the average value of all the failure theories considered, this is about a 6% deviation overall, or $\pm$3%.

<u>Conclusion and Discussion:</u>

Referring to Figure 3, the tube will fail at point A. With a yield strength of 36,300 psi, a Poisson's Ratio of 0.26, and at d equals 20in, the magnitude of 'P' that will cause failure is approximately 50 kips. Of the five different failure theories considered at this point, there is an overall deviation of about 5% regarding the value of 'P' that will cause failure. The Maximum Shear Stress Theory is the most conservative, with failure predicted when 'P' equals 49907.504 lbf. The Maximum Normal Stress Theory is the least conservative, with failure predicted when 'P' equals 52451.448 lbf. Overall, all of the considered failure theories agree quite well with one another.

This project helped to further reinforce the concepts covered in class as well as demonstrate the usefulness of solving these types of problems computationally. Being able to see a plot of the maximum magnitude of 'P' vs 'd' for each of the different failure theories aids to improve one's confidence in the results, especially when the results are in such close agreement with one another.

MATLAB Script

```
%Mason Averill
%ME-550 Fall 2020, 10/27
%Design Problem 1


%Input Parameters

%Equivalent Loads, Torques, and Moments Applied
P_axial=40;%in terms of p
T=500;%in terms of p
M_y=-1100;%in terms of p
M_z=100;%in terms of p


%Material Properties

Yield_strength=36300;%Based on A-36 steel
v=0.26;%Based on A-36 steel

d=0.1;%lower bound for d
delta_d=0.001;%increment in d per iteration

d_upper=20;%upper bound for d

size=round((d_upper-d)/delta_d+1);


p_store_tensile_max_normal_stress=zeros(1,size);

p_store_tensile_max_shear_stress=zeros(1,size);
p_store_compression_max_shear_stress=zeros(1,size);


p_store_tensile_max_normal_strain=zeros(1,size);
p_store_compression_max_normal_strain=zeros(1,size);

p_store_tensile_octahedral=zeros(1,size);
p_store_compression_octahedral=zeros(1,size);

p_store_tensile_DET=zeros(1,size);
p_store_compression_DET=zeros(1,size);


d_store=zeros(1,size);

i=1;

while(d<d_upper+delta_d)

width=2*d;
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```
height=d;
thickness=0.1*d;


%Cross-Section Properties
I_zz=2*(1/12*thickness*(height+thickness)^3)+2*(1/12*(width-
thickness)*(thickness)^3+(width-thickness)*(thickness)*(height/2)^2);

I_yy=2*(1/12*(thickness)*(width+thickness)^3)+2*(1/12*(height-
thickness)*(thickness)^3+(height-thickness)*(thickness)*(width/2)^2);

I_yz=0;

Area_mean=width*height;

Area=(width+thickness)*(height+thickness)-(width-thickness)*(height-
thickness);

%Location of Neutral Axis

argument=((M_y*I_zz-M_z*I_yz)/(M_z*I_yy-M_y*I_yz));

theta=double(atan(-argument));

theta_degrees=theta*180/pi();

%Locate Points of Max Compression and Tension


z_tension_max=-((width/2)+(thickness/2));
y_tension_max=((height/2)+(thickness/2));

z_compression_max=((width/2)+(thickness/2));
y_compression_max=-((height/2)+(thickness/2));


%Compute Stresses

%Axial Stress

Stress_axial=P_axial/Area;


%Shear Stress (same at both points of interest)

Stress_shear=T/(2*Area_mean*(thickness));

%Bending Stress

    %Max Tensile Stress

    Stress_bending_tension=(M_z*y_tension_max)/I_zz+(M_y*z_tension_max)/I_yy;
```

```
    %Max Compressive Stress

Stress_bending_compression=(M_z*y_compression_max)/I_zz+(M_y*z_compression_ma
x)/I_yy;

%Sum Stresses at Both Locations of Interest

    %Stress at Max Tensile Stress Location

    Stress_x_tension=Stress_axial+Stress_bending_tension;

    %Stress at Max Compression Stress Location

    Stress_x_compression=Stress_axial+Stress_bending_compression;

  %Now lets find principal stresses

principal_stress_max_tensile_1=(Stress_x_tension/2)+sqrt((Stress_x_tension/2)
^2+(Stress_shear)^2); %Max

  principal_stress_max_tensile_2=(Stress_x_tension/2)-
sqrt((Stress_x_tension/2)^2+(Stress_shear)^2); %Min (and less than 0)

principal_stress_max_compression_1=(Stress_x_compression/2)+sqrt((Stress_x_co
mpression/2)^2+(Stress_shear)^2); %Max

  principal_stress_max_compression_2=(Stress_x_compression/2)-
sqrt((Stress_x_compression/2)^2+(Stress_shear)^2); %Min (and less than 0)

  %Solve for P based on Maximum Normal Stress failure criterion (for brittle
%materials, i.e. tension controls)

        %At Max Tensile Location

p_solved_tensile_max_normal_stress=Yield_strength/principal_stress_max_tensil
e_1;

%Solve for P based on Maximum Shear Stress failure criterion

        %At Max Tensile Location
```

```
p_solved_tensile_max_shear_stress=Yield_strength/(principal_stress_max_tensil
e_1-principal_stress_max_tensile_2);

        %At Max Compression Location


p_solved_compression_max_shear_stress=Yield_strength/(principal_stress_max_co
mpression_1-principal_stress_max_compression_2);

%Solve for P based on Maximum Normal Strain failure criterion


        %At Max Tensile Location


p_solved_tensile_max_normal_strain=Yield_strength/(principal_stress_max_tensi
le_1-v*principal_stress_max_tensile_2);

        %At Max Compression Location


p_solved_compression_max_normal_strain=Yield_strength/(principal_stress_max_c
ompression_1-v*principal_stress_max_compression_2);


        %Solve for P based on Maximum Distortion Energy failure criterion


        %At Max Tensile Location


p_solved_tensile_DET=sqrt(2*(Yield_strength)^2/((principal_stress_max_tensile
_1-
principal_stress_max_tensile_2)^2+principal_stress_max_tensile_1^2+principal_
stress_max_tensile_2^2));

        %At Max Compression Location


p_solved_compression_DET=sqrt(2*(Yield_strength)^2/((principal_stress_max_com
pression_1-
principal_stress_max_compression_2)^2+principal_stress_max_compression_1^2+pr
incipal_stress_max_compression_2^2));


        %Solve for P based on Octahedral Shear Stress failure criterion
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```
        %At Max Tensile Location

p_solved_tensile_octahedral=sqrt(2)*Yield_strength/sqrt(((principal_stress_ma
x_tensile_1-
principal_stress_max_tensile_2)^2+principal_stress_max_tensile_1^2+principal_
stress_max_tensile_2^2));

        %At Max Compression Location


p_solved_compression_octahedral=sqrt(2)*Yield_strength/sqrt(((principal_stres
s_max_compression_1-
principal_stress_max_compression_2)^2+principal_stress_max_compression_1^2+pr
incipal_stress_max_compression_2^2));



%Store P's and d

    %Store d

    d_store(1,i)=d;

    %Store P's


p_store_tensile_max_normal_stress(1,i)=p_solved_tensile_max_normal_stress;

    p_store_tensile_max_shear_stress(1,i)=p_solved_tensile_max_shear_stress;

p_store_compression_max_shear_stress(1,i)=p_solved_compression_max_shear_stre
ss;



p_store_tensile_max_normal_strain(1,i)=p_solved_tensile_max_normal_strain;

p_store_compression_max_normal_strain(1,i)=p_solved_compression_max_normal_st
rain;

    p_store_tensile_DET(1,i)=p_solved_tensile_DET;
    p_store_compression_DET(1,i)=p_solved_compression_DET;

    p_store_tensile_octahedral(1,i)=p_solved_tensile_octahedral;
    p_store_compression_octahedral(1,i)=p_solved_compression_octahedral;



%increment d and i
```

```
d=d+delta_d;
i=i+1;

end


figure (1)
plot(d_store,p_store_tensile_max_normal_stress,'r')
xlabel('d (in)')
ylabel('Magnitude of P that will cause yielding (lbf)')
title('Magnitude of P that will cause yielding vs d: Max Tensile Location');
grid on

hold on
plot(d_store,p_store_tensile_max_shear_stress,'b')
plot(d_store,p_store_tensile_max_normal_strain,'g')
plot(d_store,p_store_tensile_DET,'k--o')
plot(d_store,p_store_tensile_octahedral,'m')
legend('Maximum Normal Stress Theory','Maximum Shear Stress Theory','Maximum
Normal Strain Theory','Distortion Energy Theory','Octahedral Theory')
xlim([0 d_upper])

hold off

figure (2)
plot(d_store,p_store_compression_max_shear_stress,'b')
xlabel('d (in)')
ylabel('Magnitude of P that will cause yielding (lbf)')
title('Magnitude of P that will cause yielding vs d: Max Compression
Location');
grid on

hold on
%plot(d_store,p_store_compression_max_normal_strain,'g')
plot(d_store,p_store_compression_DET,'k--o')
plot(d_store,p_store_compression_octahedral,'m')
legend('Maximum Shear Stress Theory','Distortion Energy Theory','Octahedral
Theory')
xlim([0 d_upper])
hold off
```