To: Professor Hosni
From: Mason Averill
Subject: ME 321 Heat Transfer Design Project: Design of a “Heat Sink”
Date: 4/14/2020

The purpose of this memo is to communicate the results of the design project for ME 321. The project consisted of designing a heat sink to be used with a silicon chip. The silicon chip had a max allowable temperature of 65°C and dissipated 1.5W at steady state. The material properties of the board the chip was mounted on and the thermal resistance at the interface between the chip and the board were both known. In addition, the convection coefficient and free stream temperature were known at all locations. To design the heat sink, two different materials were to be considered. The two different material choices were given, as well as the properties, material cost, and manufacturing cost for each material. The names of the materials were deemed “A” and “B”. The goal was to design the configuration of the heat sink as well as select the ideal material choice, such that when both were coupled all heat transfer requirements were met and the cost was minimized.

The first step to complete this project was to find how much heat transfer was possible with no heat sink, as the difference between this value and the 1.5W was the amount of heat transfer the heat sink must be capable of dissipating. Once this value was obtained, a decision about the geometry of the heat sink had to be made. The fin shape to be utilized with the heat sink was selected to be rectangular with uniform cross section. With the fin shape determined, the amount of heat transfer by the heat sink could be found generally. To optimize the heat sink, i.e. to find the configuration that met all heat transfer requirements while minimizing cost, many configurations were considered computationally, allowing for the selection of the ideal heat sink geometry and material.

The major assumptions made to complete this project were that the heat sink, chip, and board were all at steady state. In addition, the entirety of the chip was taken to be at a constant temperature of 60°C (5°C safety factor), with the temperature at the base of each/all fin(s) being the same as this temperature. The heat transfer coefficient was also taken to be constant at all locations and orientations (vertical and horizontal surfaces) for the system.

The design variables consisted of the thickness of the fin(s), the number of fins, the spacing between the fins, the width of the fins, the length of the fins, and the material of the fins.

The ideal combination turned out to be a heat sink with 3 fins of 3mm thickness each, constructed of material “A”, with width of 10mm, length of 14mm, and 0.5mm spacing between fins. The total cost of this heat sink was $0.61.

Based on the number of combinations considered, the above result is the ideal selection. However, a more ideal combination may be found if the number of combinations is increased. Choosing smaller increments for all design variables, then considering each combination of the design variables and their associated cost may yield an even cheaper heat sink.

Objective of Project:

The objective of the project was to design a heat sink capable of maintaining a silicon chip dissipating 1.5W at a maximum temperature of 65°C, while simultaneously keeping the cost of the heat sink to a minimum. The silicon chip was to be located on only the top surface of a glass/epoxy board, as shown in Figure 1. The thermal conductivity of the board and the thermal resistance at the interface between the board and chip were both known. These values were $5\ {}^{W}/_{m^2k}$ and $6.5 * 10^{-4}\ {}^{m^2k}/_{W}$, respectively. In addition, the convection coefficient and free stream temperature at all locations and surfaces were known. These values were 30 ${}^{W}/_{m^2k}$ and 25°C, respectively.

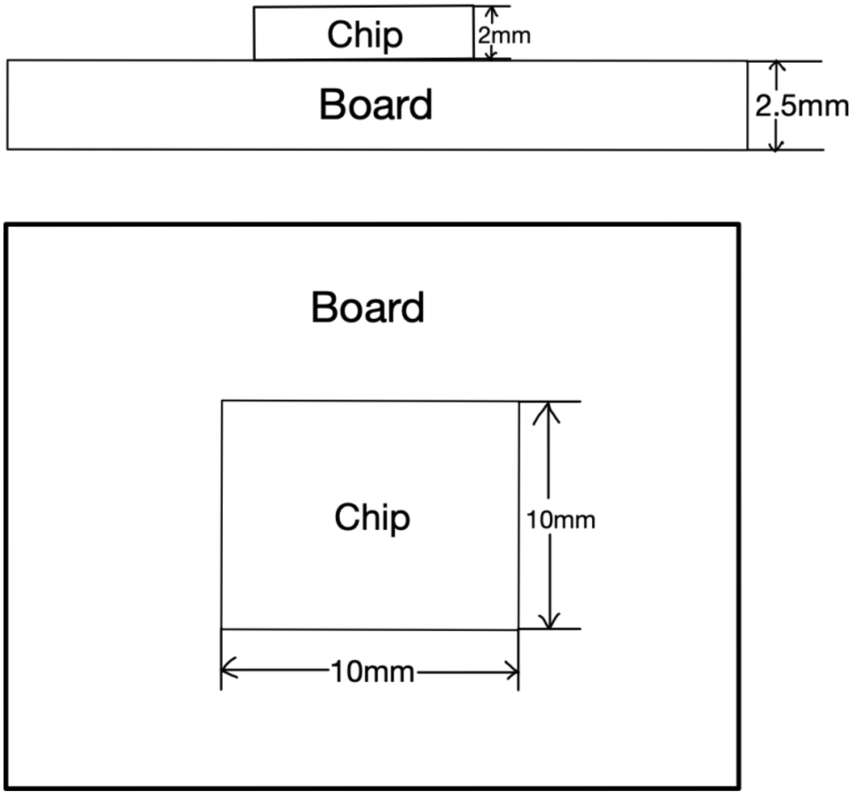


Figure 1-Chip and Board Schematic

It was also specified that two different materials were to be considered to construct the heat sink out of:

- Material A:
  ρ = 2700 kg/m^3; Cp = 0.95 kJ/kg.K; k = 230 W/m^2K; α = 97x10^{-6} m^2/s
  material cost = $6/kg
  Manufacturing cost = $[0.15 + 0.0016a$^{2.2}$] (straight or annular fins)
- Material B:
  ρ = 8800 kg/m^3; Cp = 0.387 kJ/kg.K; k = 400 W/m^2K; α = 120x10^{-6} m^2/s material cost = $10/kg
  Manufacturing cost = $[0.18 + 0.0014a$^{1.6}$] (straight or annular fins)

An overall assembly, the card cage, was to contain multiple subassemblies consisting of the glass/epoxy board, the silicon chip, and the heat sink. The subassemblies were to be located horizontally with respect to the ground in the card cage. Additionally, the subassemblies were to be positioned in a column form, such that there was a 20mm spacing between the boards in each subassembly. This configuration will be shown by a figure after the design methodology of the heat sink is discussed.

Design Methodology:

The first step to begin designing the heat sink entailed determining the rate of heat transfer the heat sink must be capable of dissipating in order to maintain the chip at a temperature of 60°C (5°C safety factor from the maximum allowable temperature of 65°C). To determine this value, the heat transfer from the bottom of the glass/epoxy board, henceforth referred to as “board”, the heat transfer from the edge of the chip, and the heat transfer from the top of the chip were found.

In order to find the heat transfer from the bottom of the board, the rate of heat conduction from the bottom surface of the chip to the bottom of the board, assuming steady state and no heat generation in the board (i.e. concept of thermal resistance applies), was set equal to the rate of heat convection at the bottom surface of the board, assuming only an area of equal size to the chip was acting in convection. The rate of heat conduction was found as shown in Equation 1. The rate of heat convection was found as shown in Equation 2.

$$\dot{Q}_{cond} = \frac{\Delta T}{\sum R_{th}} = \frac{(T_{chip} - T_{board})}{R_{board} + R_{chip-board\ interface}} = \frac{(60 - T_{board})}{\frac{Thickness_{board}}{k_{board} * A_{chip}} + 6.5 * 10^{-4}} = \frac{(60°\mathrm{C} - T_{board})}{\frac{2.5/1000}{5 * 100/10^6} + 6.5 * 10^{-4}} = \frac{(60°\mathrm{C} - T_{board})}{5.00065} \quad \text{Equation 1}$$

$$\dot{Q}_{conv} = hA_{chip}(T_{board} - T_{\infty}) = 30 * \frac{100}{10^6} * (T_{board} - 25) \quad \text{Equation 2}$$

Setting Equation 1 equal to Equation 2 and solving yields $T_{board} \cong 59.48$°C. Plugging this value into either Equation 1 or Equation 2 yields $\dot{Q} \cong 0.103W$. This is only the rate of heat transfer from the bottom of the chip. To find the rate of heat transfer from the side of the chip, Equation 3 was utilized, again operating under the assumption that the entire chip was of uniform temperature.

$$\dot{Q}_{conv-chip\ sides} = hA_{chip\ sides}\left(T_{chip} - T_{\infty}\right) = 30 * \frac{4 * 2 * 10}{10^6} * (60 - 25) = 0.084W \quad \text{Equation 3}$$

Next, the heat transfer from the top of the chip was found, assuming that there was no heat sink. This was found as shown by Equation 4.

$$\dot{Q}_{conv-chip\ top} = hA_{chip\ top}\left(T_{chip} - T_{\infty}\right) = 30 * \frac{10 * 10}{10^6} * (60 - 25) = 0.105W \qquad \text{Equation 4}$$

The total heat transfer from the bottom, sides, and top of the chip was thus found to be $0.103W + 0.084W + 0.105W = 0.292W$. Since the chip is known to generate 1.5W at steady state, the heat sink must be capable of dissipating 1.5W-0.292W=1.208W.

Next, with the rate of heat transfer that the heat sink must be capable of dissipating known, it was time to choose some general properties of the heat sink. It was decided that the heat sink would have a fin(s) of constant rectangular cross section, with heat dissipation from the end of the fin(s). Utilizing Equation 3.77 from Bergman[1] the total rate of heat transfer from a single fin was found, as shown by Equation 5.

$$\dot{Q}_{conv/fin} = M \frac{\sinh(mL) + \left({}^{h}/_{mk}\right) \cosh(mL)}{\cosh(mL) + \left({}^{h}/_{mk}\right) \sinh(mL)} \qquad \text{Equation 5}$$

With:
M= $\sqrt{hPkA_c}\theta_b$
θ_b=$T_b - T_{\infty}$
m=$\sqrt{\frac{hP}{kA_c}}$
P=Perimeter of cross section
A_c=fin cross sectional area

Figure 2 shows a top and side view of the heat sink. Figure 3 shows a top and side view of the board, chip, and heat sink assembly. Figure 4 shows the convection surfaces and the properties of the heat conduction in the board (i.e. only 1-D). Figure 5 shows the configuration of the card cage.

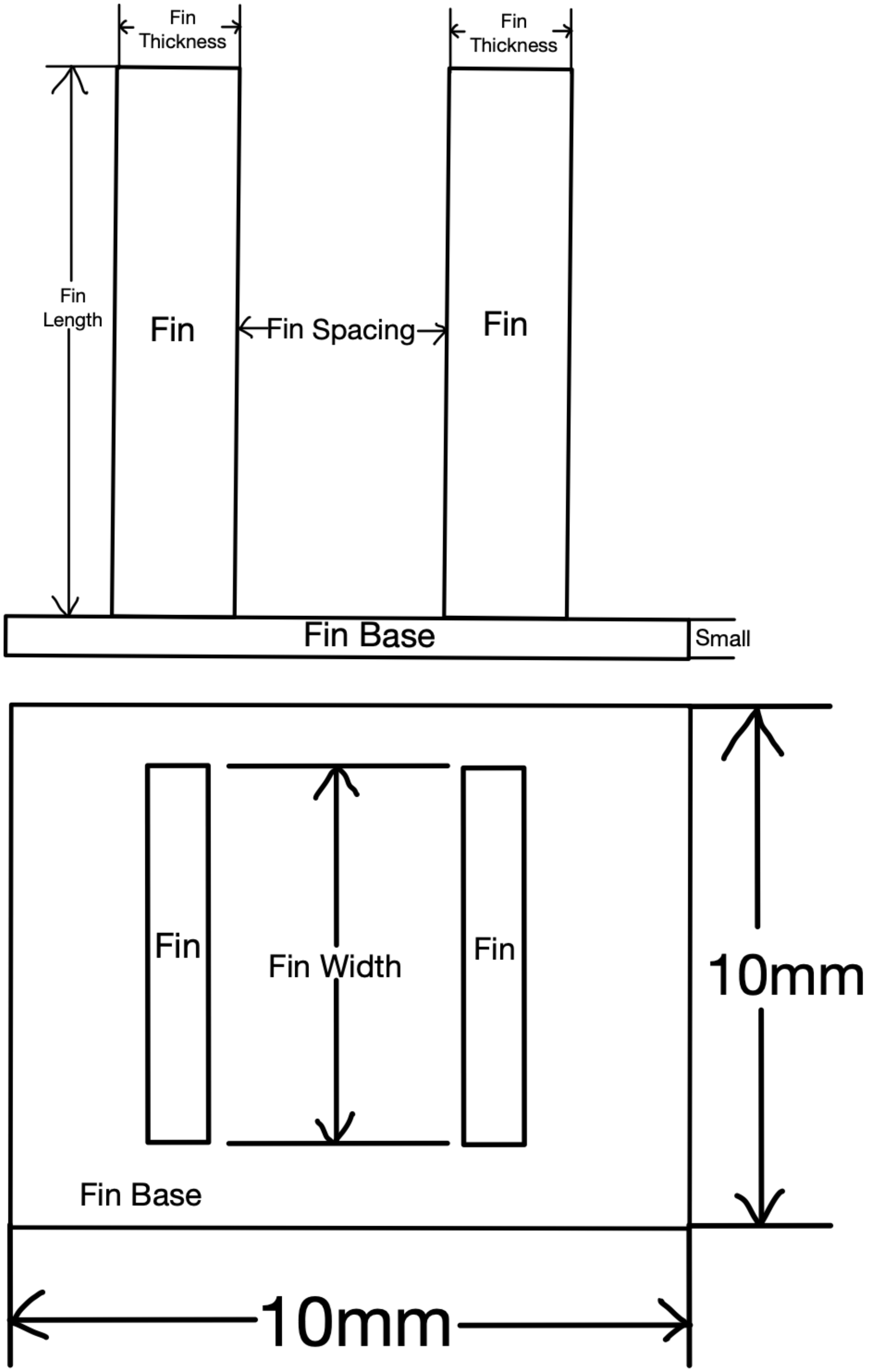


Figure 2-Top and Side View of the Heat Sink

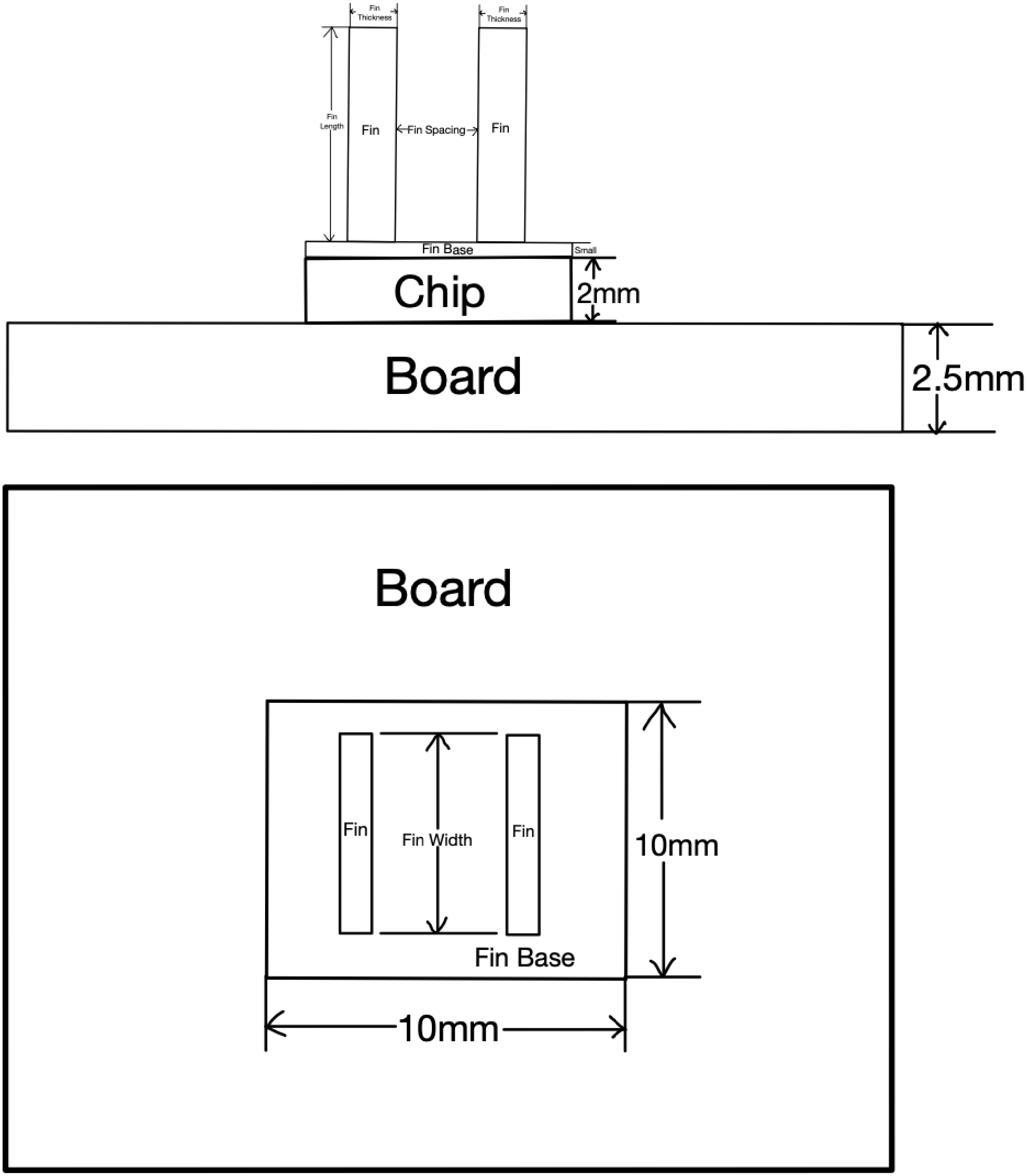


Figure 3-Top and Side View of Board, Chip, and Heat Sink Assembly

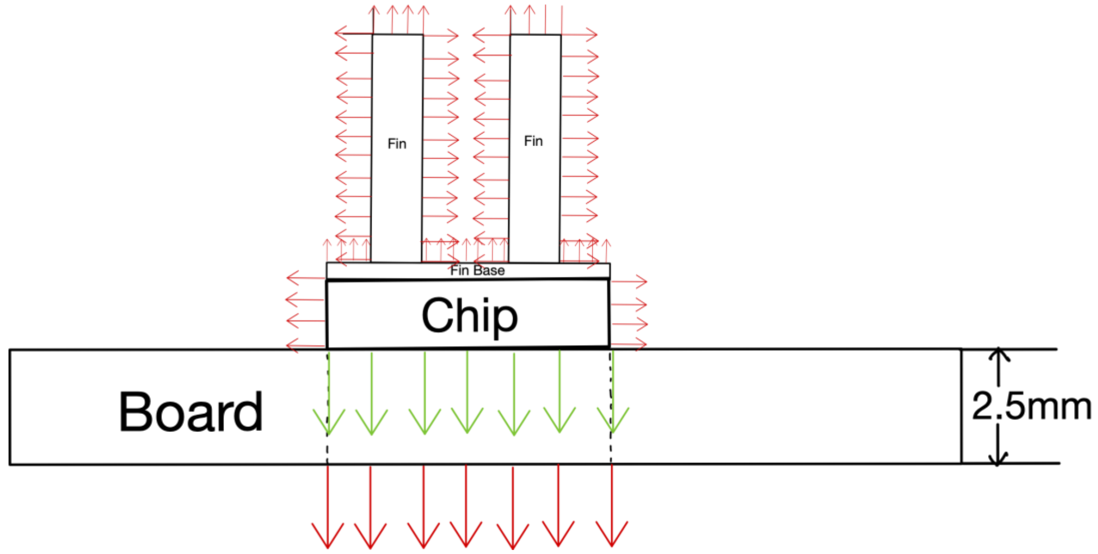


Figure 4-Modes of Heat Transfer (Red=Convection, Green=Conduction)

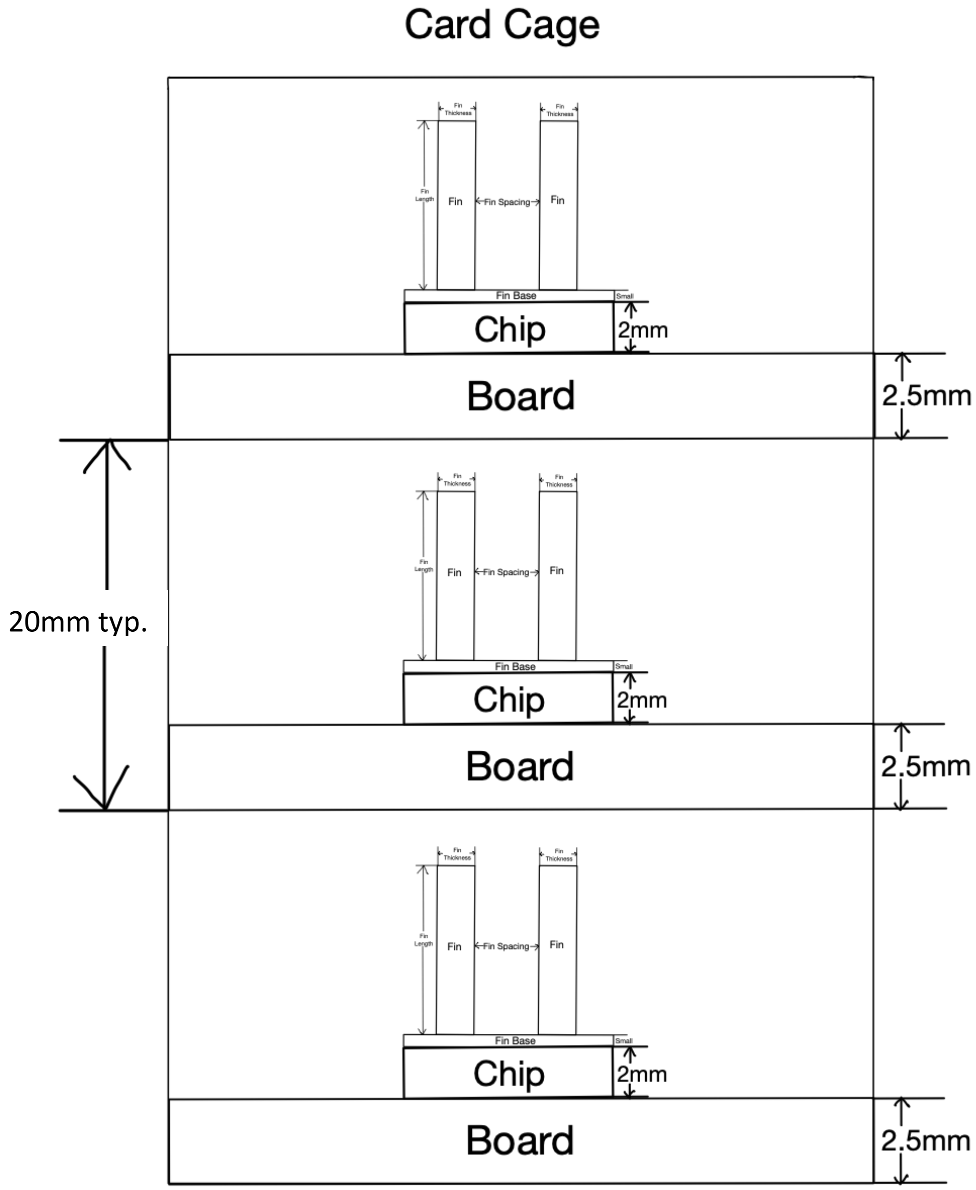


Figure 5-Card Cage Configuration

In order to determine which specific combination of fin thickness, number of fins, fin width, fin length, spacing between fins, and fin material yielded the lowest cost, computational methods were utilized. Fin thicknesses from the minimum allowable value of 0.5mm to the maximum allowable value, based on the dimension of the chip, of 10mm were considered in 0.25mm increments. The number of fins considered ranged from 1 to 20, at increments of 1. The width of the fin(s) was user-specified. It was determined that for brevity of investigation the width would be held constant at 10mm, meaning that the fin would span over the entire chip's width. The fin length was also a user-specified value, the values of length considered ranged from 1mm to 15mm in 1mm increments. The span between fins was another user-specified value. Without knowing the minimum allowable distance between fins from a manufacturing perspective, it was determined that the span would be set to 0.5mm, as this was the minimum allowable width of the fin from the manufacturers. In total, 11,400 possible combinations were considered for each material type.

The following order of computations was utilized to analyze all possible combinations:

1) A matrix consisting of each possible combination of fin thickness and number of fins was created. A set of constraints was developed to determine whether or not each combination was capable of occurring given the size of the chip and the spacing between fins. A value of 1 was assigned to the combinations that were possible, a value of 0 was assigned to those that were not possible.

2) Of the combinations that were dimensionally possible as determined from step 1, the perimeter and cross sectional area was found for each fin thickness, given a user specified fin width.
3) Referring to Equation 5, m and M were found for each possible combination as determined from step 1, for both materials A and B.
4) The heat transfer due to the fins was determined for each possible combination, as determined from step 1, given a user-specified input of length.
5) The total heat transfer was found for each possible combination by adding the following heat transfers:
    a. HT from the bottom of the board, see the result of Equation 1 and 2
    b. HT from the edge of the chip, see Equation 3
    c. HT from the top of the chip, see Equation 4 (note that the total cross-sectional area of the fin(s) was subtracted from the surface area of the top of the chip, also note that the base of the fins was of small thickness and had a negligible thermal resistance)
    d. HT from the heat sink/fins
6) The material cost was determined for each possible combination, as determined from step 1.
7) The manufacturing cost was determined for each possible combination, as determined from step 1.
8) The total cost was found by combining the result of steps 6 and 7.

9) A final matrix consisting of all possible combinations was generated that checked to see if the following conditions were met:
    a. The combination was physically possible, as determined from step 1. If not a value of 0 was assigned. If it was a possible configuration the next check was initiated
    b. The combination was checked to ensure that the total heat transfer was greater than or equal to 1.5W. If not a value of 0 was assigned. If it was a possible configuration the next step was initiated
10) If both 9a and 9b conditions were met, then the final matrix populated the appropriate combination with its total cost, as determined from step 8.
11) A conditional formatting was applied. The formatting performed the following actions:
    a. All possible combinations from Step 10 were highlighted in yellow
    b. The minimum-cost combination was highlighted in green

Major Assumptions:

The major assumptions made have been discussed as necessary when explaining the design methodology, however, the following list compiles all major assumptions made into a more compact form:

1) The heat sink, chip, and board were all at steady state
2) The entirety of the chip was taken to be at a constant temperature of 60°C (5°C safety factor)

3) The temperature at the base of each fin was taken to be the same as the chip's temperature
4) The thermal resistance due to the fin's base was considered small. The thickness of the heat sink base was also considered small.
5) The convection coefficient was taken to be constant at all locations and orientations
6) The heat conduction in the board was 1-D, and the heat convected from the board was based on an area of equal size as the bottom surface area of the chip.
7) The fins were taken to be rectangular and of constant cross section, with heat transfer at the tip of the fin.

Design Variables:

The design variables have been discussed as necessary when explaining the design methodology, however, the following list compiles all design variables into a more compact form:

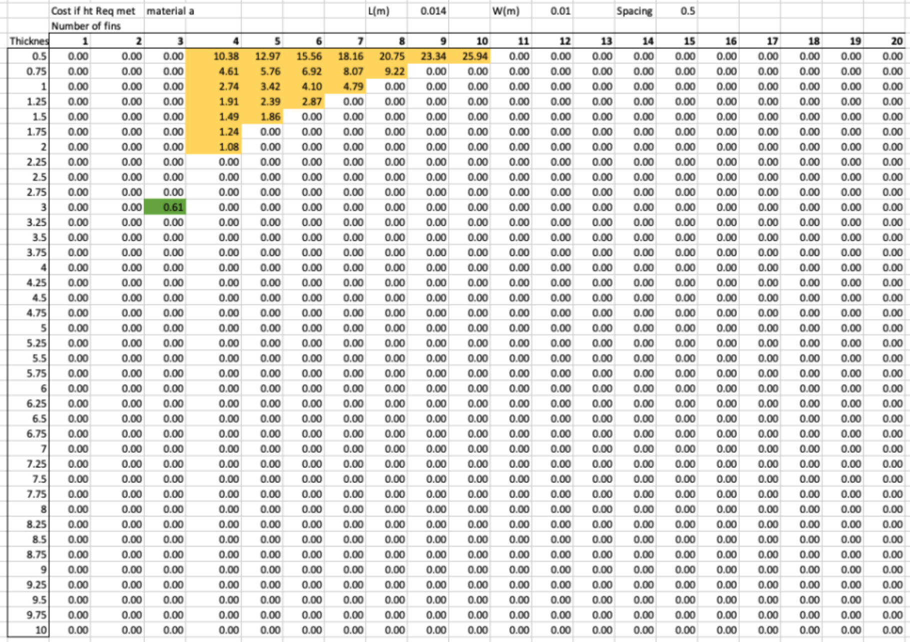
1) Fin thickness
2) Number of fins
3) Spacing between fins
4) Width of the fins
5) Length of the fins
6) Material of the fins

Results:

After 22,800 combinations of the aforementioned 6 design variables were considered, the ideal combination consisted of the following properties:

- Fin thickness=3mm
- Number of fins=3
- Spacing between fins=0.5mm
- Width of the fins=10mm
- Length of the fins=14mm
- Material of the fins=Material A

This particular combination yielded a total heat transfer of approximately 1.52W and costed just $0.61 for the entire heat sink. Figure 6 shows this result as the output of the computational system used.



	Cost if ht Req met		material a					L(m)	0.014		W(m)	0.01		Spacing	0.5					
	Number of fins																			
Thicknes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.5	0.00	0.00	0.00	10.38	12.97	15.56	18.16	20.75	23.34	25.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.75	0.00	0.00	0.00	4.61	5.76	6.92	8.07	9.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	0.00	0.00	0.00	2.74	3.42	4.10	4.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.25	0.00	0.00	0.00	1.91	2.39	2.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.5	0.00	0.00	0.00	1.49	1.86	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.75	0.00	0.00	0.00	1.24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	1.08	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figure 6-Optimized Heat Sink Combination

Conclusions and Recommendations:

By coupling an understanding of heat transfer concepts and computational methods, an optimized solution was found for this design project. In the event that this project's methodology was to be applied to a real application, it would be a good idea to consider even more possible combinations. While the cost may not be able to be reduced significantly past what was found for this project, even a savings of only a few cents per heat sink would be significant if the production volume was to be large. In order to accomplish this, it would be recommended to construct a MATLAB code rather than the iterative-method used for this project. This would allow for much smaller increments in the design variable's values without costing a lot of computational time.

References

1. Bergman, T. L. (2015). *Introduction to heat transfer: Thrm 320*. Jefferson City: Wiley.