Cooling Subsystem

Temperature Distribution Hand Calculations:

The cooling subsystem consists of five fans covered by a 3-D printed duct that lie below the extruded plastic and cause forced convection to cool the hot filament.

The first design decision of the cooling subsystem was to determine its maximum length. After consideration as a group, a length of 600 mm, or 0.6 m, was assumed to be a sufficient enough length that would not hinder the feasibility of the desired length of the entire assembly. Next, an analytical equation of the temperature distribution of the filament extruded from the nozzle was developed. Five assumptions were made before doing so. The situation represents a quasi-steady, one-dimensional conduction problem with no internal heat generation and no radiation, while the boundary of the filament is losing heat to convection. Assuming the situation to be quasi-steady allowed for a one-dimensional temperature distribution along the axial direction of the extruding filament. This temperature distribution equation is displayed below in Equations (1) and (2).

$$T(x) = (T_{exit} - T_\infty)e^{Rx} + T_\infty \tag{1}$$

Where,

$$R = \frac{1}{2}\left[\frac{\dot{m}c_p}{A_c k} - \sqrt{\left(\frac{\dot{m}c_p}{A_c k}\right)^2 + \frac{16h_c}{kD}}\right] \tag{2}$$

The temperature distribution can be used to get an analytical solution for amount of heat lost due to convection for any selected length. This equation is displayed below in Equation (3). This value can also be verified using the specific heat formula that describes the amount of heat required to change the temperature of the material a finite amount.

$$\dot{Q}_{loss} = \frac{2(T_{exit} - T_\infty)\pi D h_c}{R}\left[e^{RL} - 1\right] \tag{3}$$

The only fixed parameter in the equations is the diameter of the filament, D, which is specified at 1.75 mm. This diameter related directly to the cross-sectional area, A_c. Assuming a constant density of the filament, the three material properties of the plastic that affect the temperature distribution are the extrusion temperature, T_{exit}, the specific heat, c_p, and the thermal conductivity, k. Finally, the mass flow rate, $\dot{m}$, was assumed to be 1 kg/hour, since this is likely the fastest that the extruder will be pushed to achieve, and the calculations were completed with the harshest possible conditions.

This left the heat transfer coefficient, h_c and the ambient temperature, T_∞, as independent variables of the system that could be altered to achieve the desired effects. Temperatures of 60°C or below are considered to be fully cooled. This temperature allows for all of the plastics that will be extruded to be fully hardened without any malleability.

If the temperature of the ambient air is not being altered in any way, then it can be assumed to be approximately 20 °C. Using this information, a minimum heat transfer coefficient required to cool the filament to the cooling temperature by 0.6 meters can be calculated for each material. Table 1 and Table 2 below outline the material properties of PLA and ABS, and the corresponding required convection coefficient, heat removed by convection for the convection coefficient, and the theoretical heat loss required to cool the material to the desired temperature.

Table 1: Material Properties for PLA and ABS

Material	Specific Heat (J/kgC)	Thermal Conductivity (W/mK)	Extrusion Temperature (°C)
PLA	1800	0.13	190
ABS	1920	0.26	260

Table 2: Required heat transfer coefficient and heat loss for each material.

Material	Heat Transfer Coefficient (W/m^2K)	Required Heat Loss (W)	Heat Loss from Convection (W)
PLA	219	65	64.96
ABS	289.5	106.667	106.6313

It can be observed that the heat transfer coefficient for ABS is required to be about 290 W/m^2K, while the coefficient for PLA is roughly 219 W/m^2K. The filament will also lose heat due to radiation, approximately 10 W/m^2K. This can be subtracted from the overall heat transfer coefficient to give the minimum required convection coefficient required by the forced convection apparatus.

Convection Coefficient Hand Calculations:

Now that the minimum convection coefficient was determined, a fan system that could output this onto the filament needed to be designed. The convection coefficient over a cylinder in cross flow is difficult to precisely calculate, since there are multiple equations that are accepted in fluid mechanics. Since the only controllable variable of the convection coefficient was the free stream velocity, multiple accepted equations for the convection coefficient over a cylinder in cross flow were used to create a range of convection coefficients that the system would portray. Equations (4) and (5) are the formulas used to determine the range of values.

$$\frac{\bar{h}D}{k} = 0.3 + \frac{0.62Re_D^{1/2}Pr^{1/3}}{[1+\left(\frac{0.4}{Pr}\right)^{\frac{2}{3}}]^{1/4}}\left[1+(\frac{Re_D}{282{,}000})^{5/8}\right]^{4/5} \tag{4}$$

$$\frac{\bar{h}D}{k} = CRe_D^{m}Pr^{0.37}\left(\frac{Pr}{Pr_s}\right)^{1/4} \tag{5}$$

Table 3: Values of C and m for Equation (5).

Re_D	C	m
0.4-4	0.989	0.33
4-40	0.911	0.385
40-4,000	0.683	0.466
4,000-40,000	0.193	0.618
40,000-400,000	0.027	0.805

Based on these two equations, the only controllable variable is the free stream velocity embedded inside of Reynold's number. Using both of these equations, a plot can be formed showing the velocity's effect on the average heat transfer coefficient across the cylinder to give a range of convection coefficients based on any given free stream velocity.

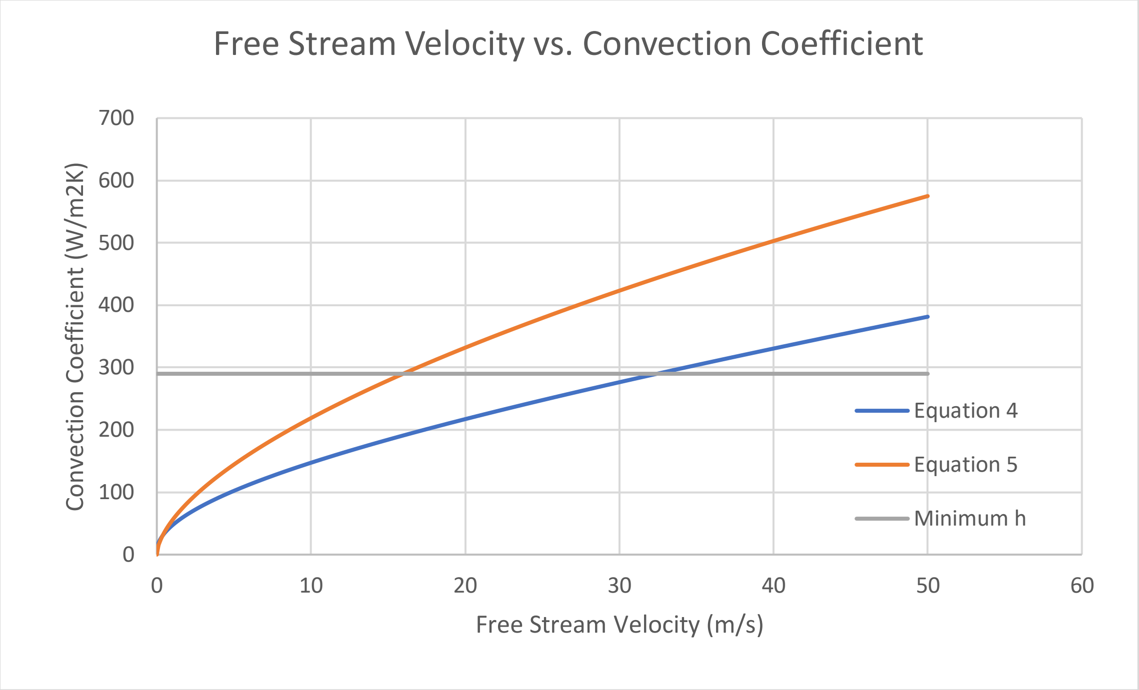


Figure 1: Free stream velocity vs. convection coefficient using both Nusselt number equations.

Based on Figure 1, it can be determined that the absolute minimum air velocity that the fans must be able to produce over a 600 mm span is roughly 30 m/s. When designing the fans, there needs to be a margin for error, so the apparatus should be capable of producing a 40 m/s air velocity.

Using the CFM rating of a fan, the free stream velocity can be determined from the given volumetric flow rate and a controlled cross-sectional area. For the 600 mm span, it was determined that five 120 x 120 mm axial cooling fans capable of 115 CFM flow rates side by side with a tapered duct secured on top of them would produce the sufficient air velocity onto the filament. A duct was then created in SolidWorks and displayed below in Figure 2.

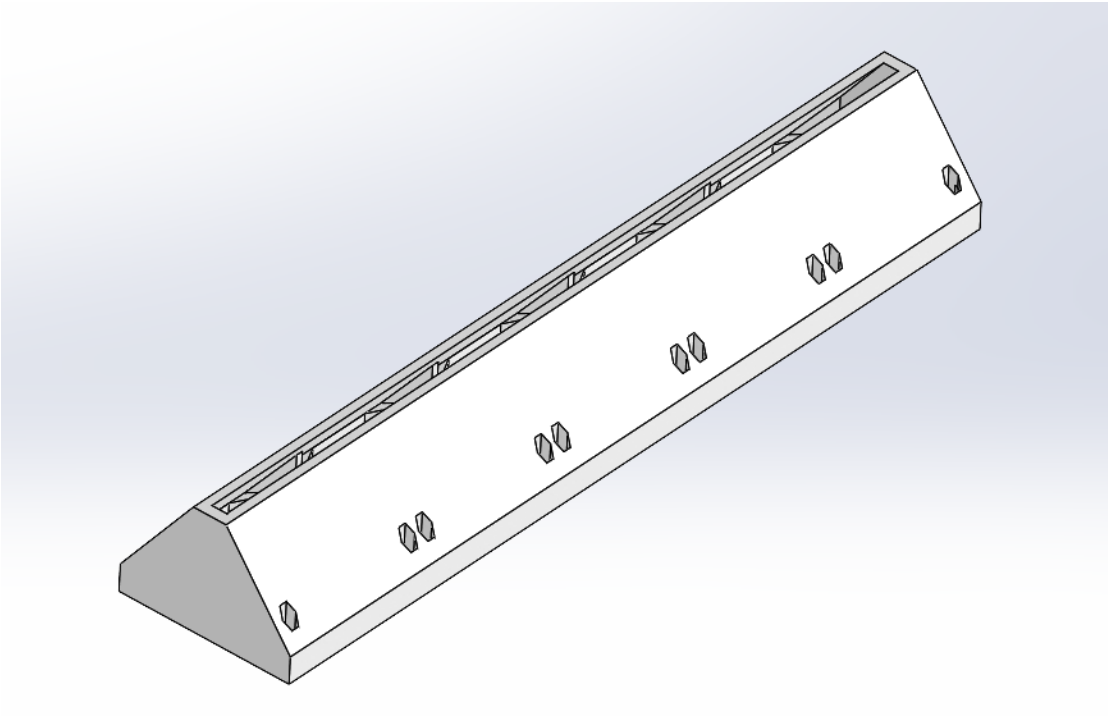


Figure 2: Isometric View of Duct Attachment

This duct has an exit cross section of 11.21 x 593 mm, allowing for the 575 CFM of air to be sped up to 40.79 m/s. If a perforated plate is attached to the top of the duct, the velocity will be capable of even higher speeds. The duct also has holes in the top face that will allow for guides created out of 1.5 mm wire to be inserted into and have the filament slide along the path.

Ensuring that the filament is fully cooled before entering the spooling process is essential to prevent any possible defects due to malleability of the material. Using the maximum possible power input to the fans, they produce 575 CFM, which equates to an air velocity of 40.79 m/s. Using Equations (4 and (5, the average convection coefficients equate to a range of 334.6-508.9 W/m^{2}K. Plugging the lower bound of this convection coefficient into the analytical temperature distribution equation yields Figure 3 below, assuming ABS plastic. The heat that is required to be removed from the filament between the nozzle and the 0.6 m for ABS (for a temperature difference of 200 °C) equates to 106.667 W, while the heat removed purely by convection equates to 111.8 W over the span of the cooling subsystem.

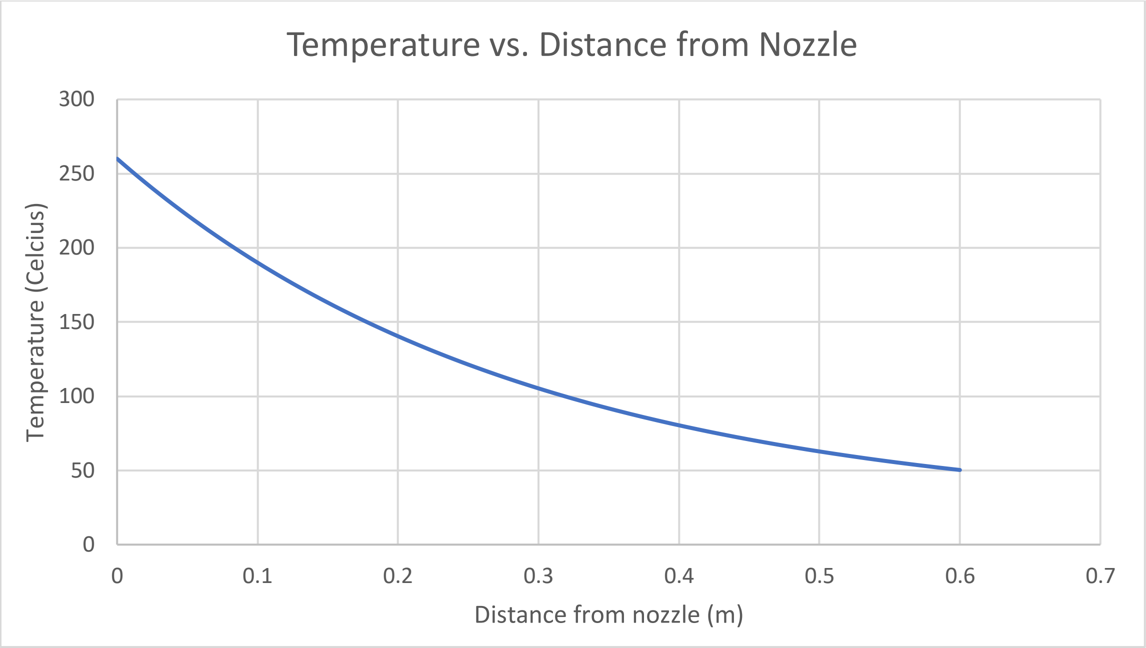


Figure 3: Temperature distribution of ABS undergoing a convection coefficient of 334.6 W/m^{2}K.

Currently these calculations neglect the positive effects that radiation would have on the cooling of the filament, but a rough estimate using radiation principles approximates that for ABS, the heat transfer coefficient due to radiation is 9.775 W/m^{2}K.

The length at which the filament reaches the desired cooled temperature of 60 °C is 0.519 m, well before the desired 0.6 m.

ABS plastic is the material that will require the most fan speed, and the lower bounds of the system easily satisfy the requirements. At this point in the design process, the cooling subsystem takes the configuration of Figure 4 below (note that the filament's small size makes it difficult to view).

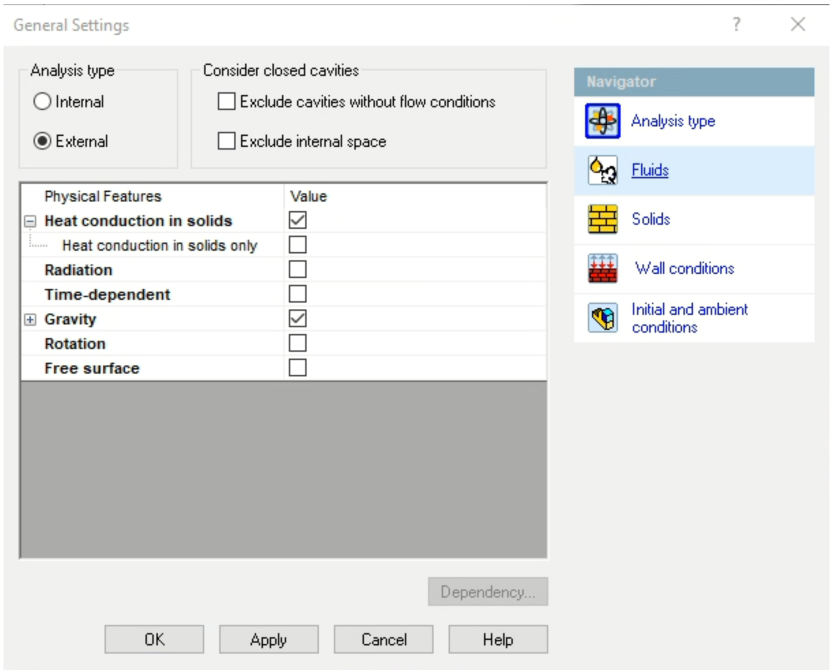
Figure 4: Overview of Cooling Sub-System

SolidWorks CFD Verification of the Temperature Distribution and Heat Transfer Coefficient:

In order to validate the analytical solution methods, a Solidworks flow simulation was created. The simulation was inclusive of both the velocity of the free stream air due to the forced convection from the fans as well as the translational motion of the filament across the cooling section. To capture both of these qualities in a single simulation, the following situation was considered:

- An external analysis type was selected
- Two fluid domains were established, one for the ambient air and one for the filament material (polyethylene was considered)
- A very thin jacket was created to bound the filament. This was necessary to include the translational motion of the filament across the cooling section. The jacket's wall thickness was very small and had a large thermal conductivity (pure silver was selected)
- To minimize the effects of the bounding jacket on the filament's velocity profile, a wall motion boundary condition was set at the interior of the jacket equivalent to that of the inlet velocity of the filament. In addition, a slip condition was added at the boundary between the filament and the jacket
- A volume flow rate was prescribed at the bottom of the duct equivalent to that of all the fan contributions

Figure 5 - Figure 7 depict these simulation parameters.



General Settings

Analysis type

○ Internal

● External

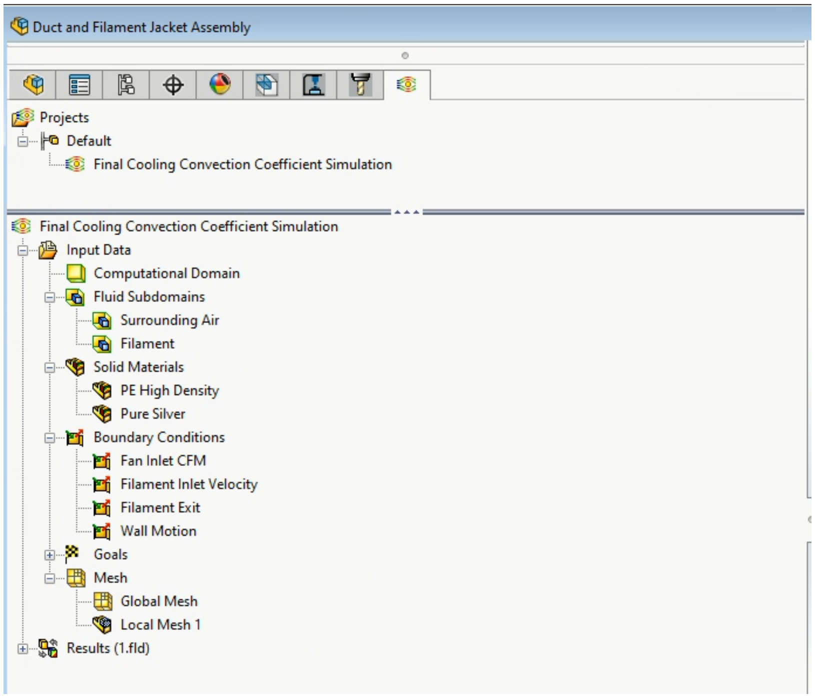
Consider closed cavities

☐ Exclude cavities without flow conditions

☐ Exclude internal space

Physical Features	Value
Heat conduction in solids	☑
Heat conduction in solids only	☐
Radiation	☐
Time-dependent	☐
Gravity	☑
Rotation	☐
Free surface	☐

Figure 5: General Setting for Flow Simulation



Duct and Filament Jacket Assembly

- Projects
  - Default
    - Final Cooling Convection Coefficient Simulation

- Final Cooling Convection Coefficient Simulation
  - Input Data
    - Computational Domain
    - Fluid Subdomains
      - Surrounding Air
      - Filament
    - Solid Materials
      - PE High Density
      - Pure Silver
    - Boundary Conditions
      - Fan Inlet CFM
      - Filament Inlet Velocity
      - Filament Exit
      - Wall Motion
    - Goals
    - Mesh
      - Global Mesh
      - Local Mesh 1
  - Results (1.fld)

Figure 6: Input Data for Flow Simulation

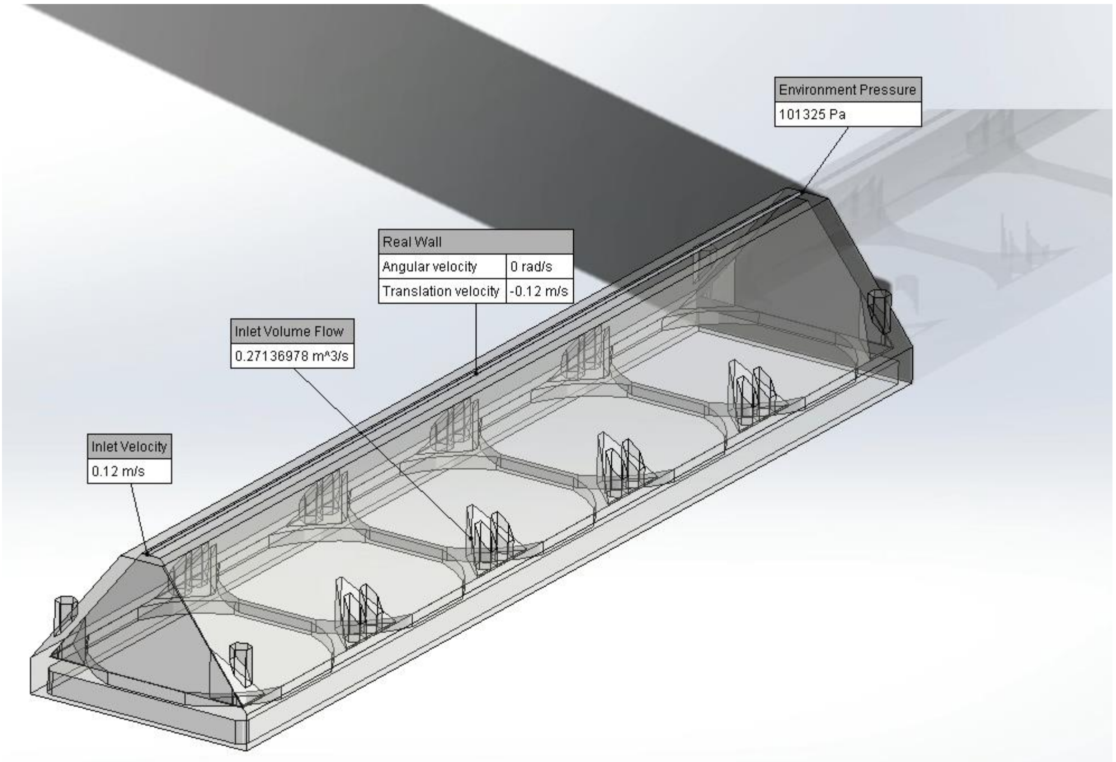
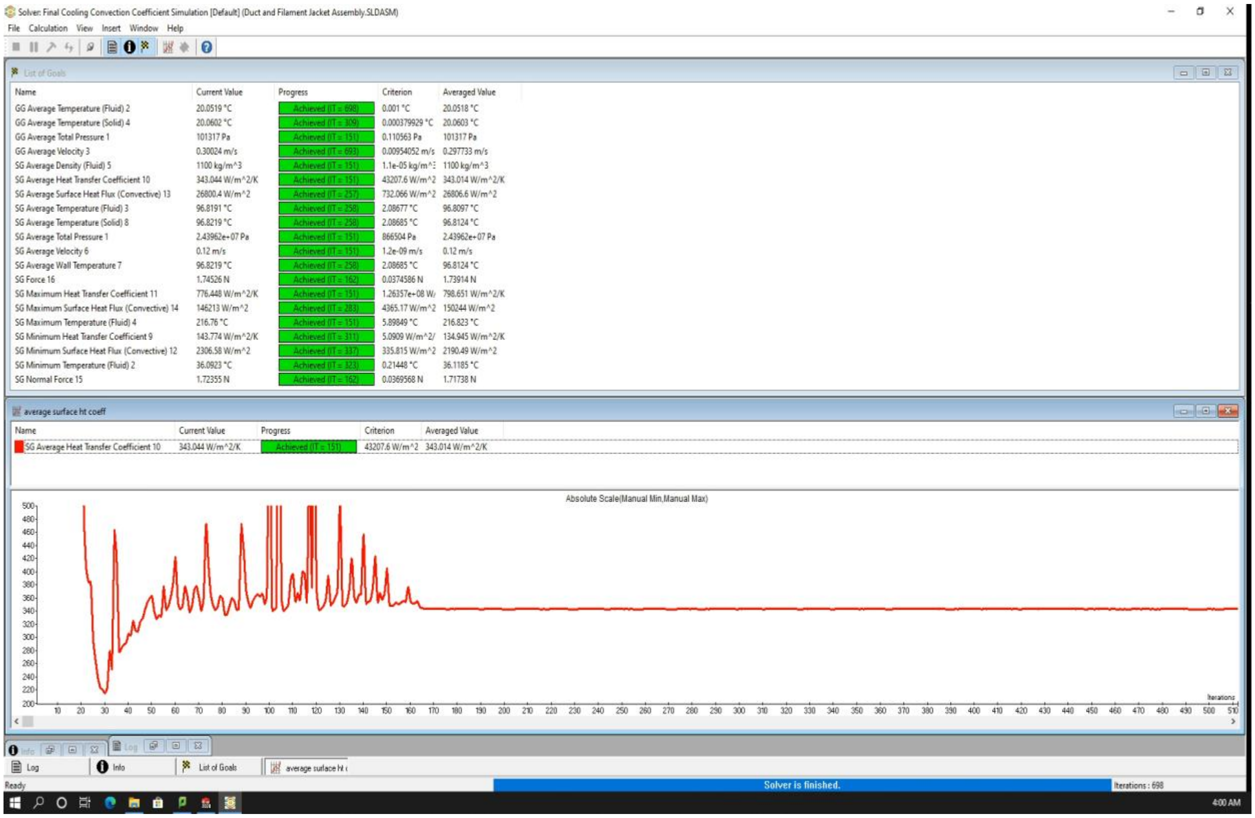


Figure 7: Boundary Conditions for Flow Simulation

The simulation was then executed and solved; Figure 8 indicates the convergence qualities of the goals considered.



Name	Current Value	Progress	Criterion	Averaged Value
GG Average Temperature (Fluid) 2	20.0519 °C	Achieved (IT = 698)	0.001 °C	20.0518 °C
GG Average Temperature (Solid) 4	20.0602 °C	Achieved (IT = 309)	0.000379929 °C	20.0603 °C
GG Average Total Pressure 1	101317 Pa	Achieved (IT = 151)	0.110563 Pa	101317 Pa
GG Average Velocity 3	0.30024 m/s	Achieved (IT = 693)	0.00954052 m/s	0.297733 m/s
SG Average Density (Fluid) 5	1100 kg/m^3	Achieved (IT = 151)	1.1e-05 kg/m^3	1100 kg/m^3
SG Average Heat Transfer Coefficient 10	343.044 W/m^2/K	Achieved (IT = 151)	43207.6 W/m^2	343.014 W/m^2/K
SG Average Surface Heat Flux (Convective) 13	26800.4 W/m^2	Achieved (IT = 257)	732.066 W/m^2	26806.6 W/m^2
SG Average Temperature (Fluid) 3	96.8191 °C	Achieved (IT = 258)	2.08677 °C	96.8097 °C
SG Average Temperature (Solid) 8	96.8219 °C	Achieved (IT = 258)	2.08685 °C	96.8124 °C
SG Average Total Pressure 1	2.43962e+07 Pa	Achieved (IT = 151)	866504 Pa	2.43962e+07 Pa
SG Average Velocity 6	0.12 m/s	Achieved (IT = 151)	1.2e-09 m/s	0.12 m/s
SG Average Wall Temperature 7	96.8219 °C	Achieved (IT = 258)	2.08685 °C	96.8124 °C
SG Force 16	1.74526 N	Achieved (IT = 162)	0.0374586 N	1.73914 N
SG Maximum Heat Transfer Coefficient 11	776.448 W/m^2/K	Achieved (IT = 151)	1.26357e+08 W/	798.651 W/m^2/K
SG Maximum Surface Heat Flux (Convective) 14	146213 W/m^2	Achieved (IT = 283)	4365.17 W/m^2	150244 W/m^2
SG Maximum Temperature (Fluid) 4	216.76 °C	Achieved (IT = 151)	5.89849 °C	216.823 °C
SG Minimum Heat Transfer Coefficient 9	143.774 W/m^2/K	Achieved (IT = 311)	5.0909 W/m^2/	134.945 W/m^2/K
SG Minimum Surface Heat Flux (Convective) 12	2306.58 W/m^2	Achieved (IT = 337)	335.815 W/m^2	2190.49 W/m^2
SG Minimum Temperature (Fluid) 2	36.0923 °C	Achieved (IT = 323)	0.21448 °C	36.1185 °C
SG Normal Force 15	1.72355 N	Achieved (IT = 162)	0.0369568 N	1.71738 N

Name	Current Value	Progress	Criterion	Averaged Value
🟥 SG Average Heat Transfer Coefficient 10	343.044 W/m^2/K	Achieved (IT = 151)	43207.6 W/m^2	343.014 W/m^2/K

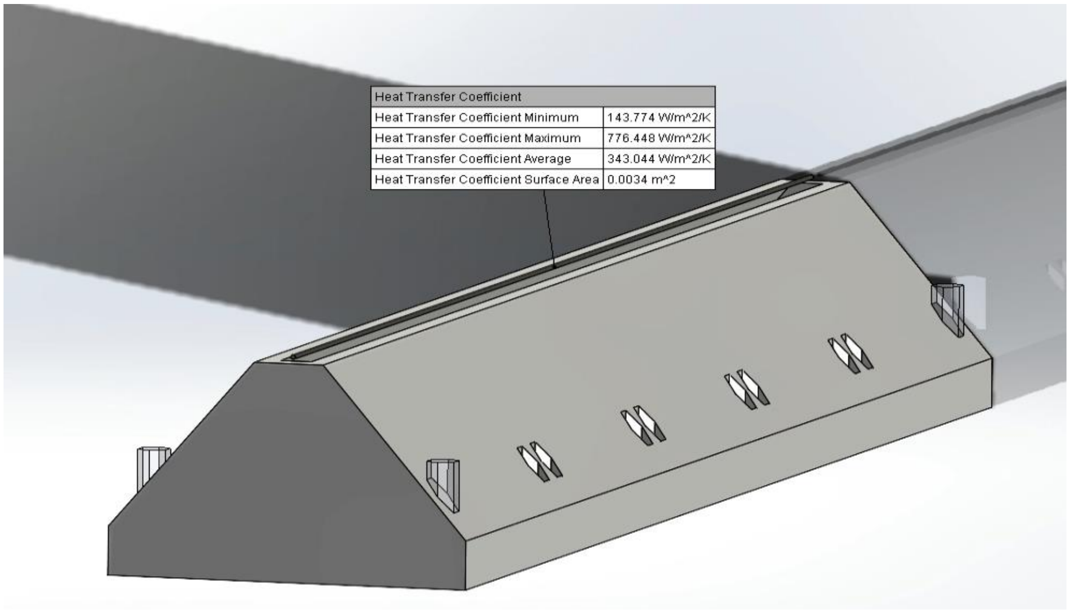
Ready

Solver is finished.

Iterations : 698

Figure 8: Convergence Data for Flow Simulation Solver

Once the simulation was successfully solved, the convective heat transfer coefficient at the outer surface of the jacket (equivalent to the filament) was displayed. This is shown by Figure 9. Note that the average convective coefficient value matches that of Figure 8, as it should, but also displays the minimum and maximum values across the surface.



Heat Transfer Coefficient	
Heat Transfer Coefficient Minimum	143.774 W/m^2/K
Heat Transfer Coefficient Maximum	776.448 W/m^2/K
Heat Transfer Coefficient Average	343.044 W/m^2/K
Heat Transfer Coefficient Surface Area	0.0034 m^2

Figure 9: Heat Transfer Coefficient

It is important to note that the average convection coefficient determined from the simulation was 343 W/m^{2}K. By reviewing Figure 9, it is evident that this value falls between the analytical solution methods considered. It most nearly matches the value expected from Equation (5), with a percent error of just 2.5%. The velocity distribution at the top of the duct is shown by Figure 10 - Figure 11. Note that this velocity distribution matches very well with the analytical prediction, which was an average of 40.8 m/s.

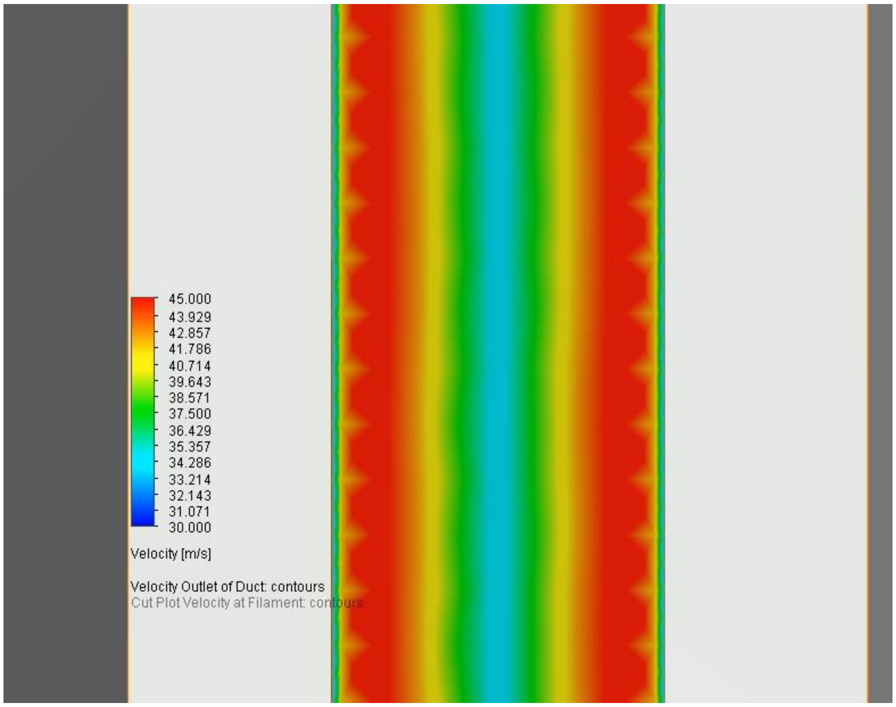


Figure 10: Velocity Profile at the Top of the Duct

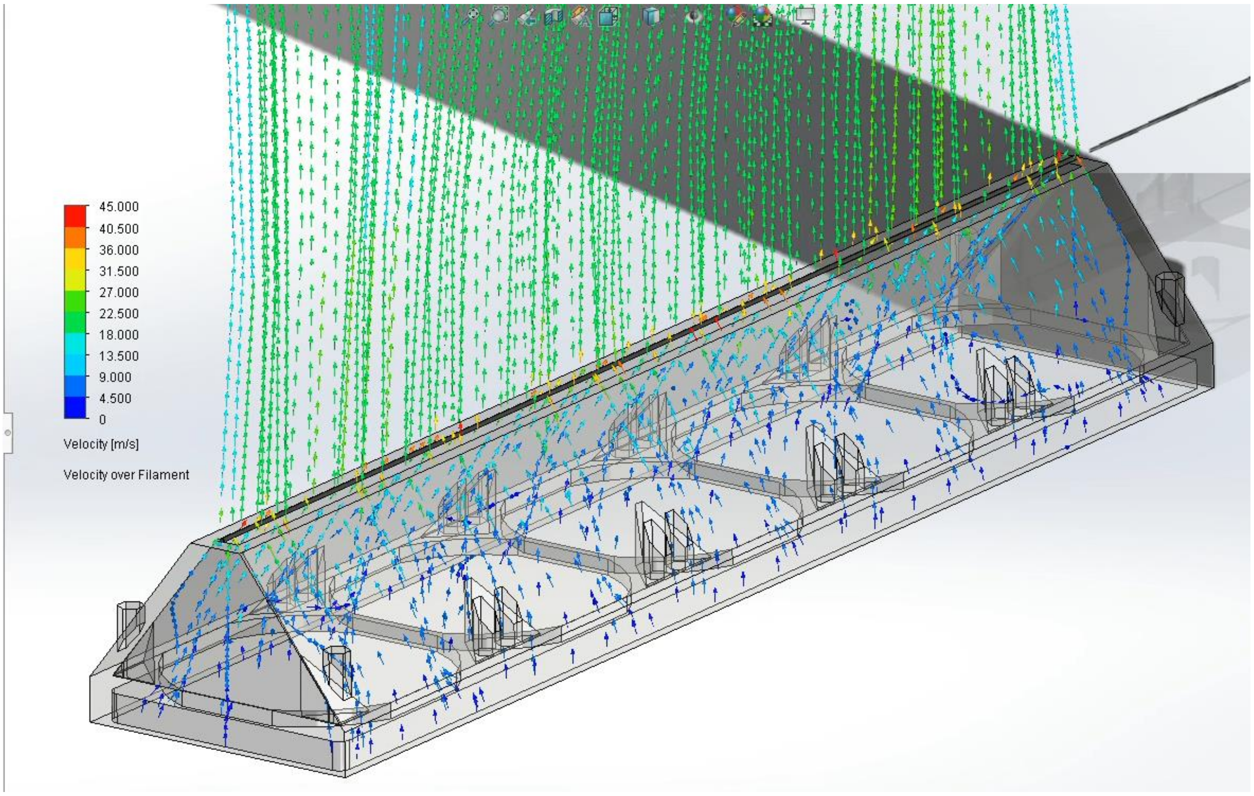
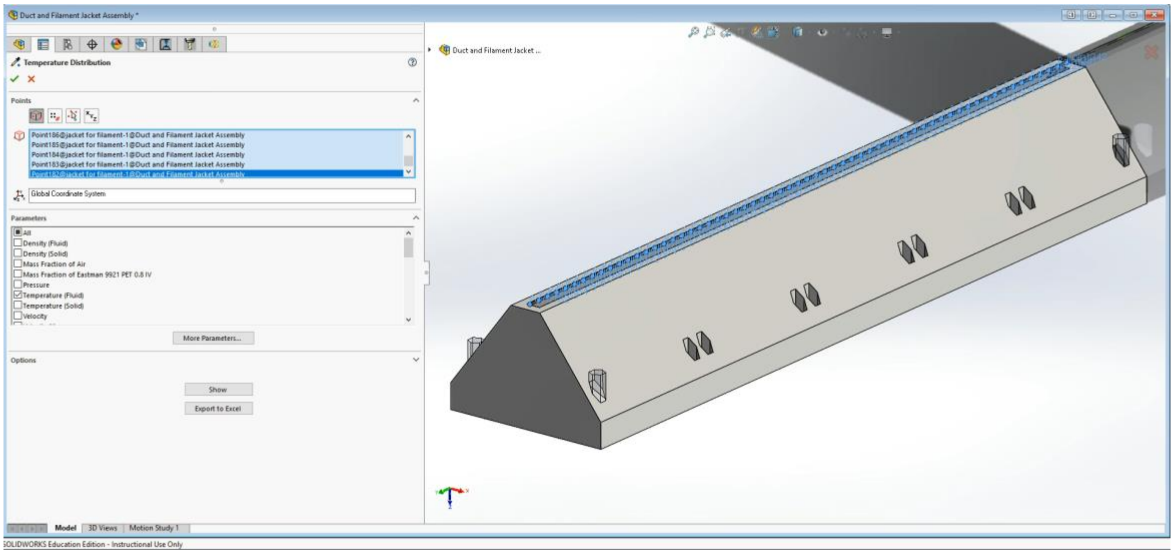


Figure 11: Velocity Trajectories Across the Filament and Inside the Duct

To compare the temperature distribution of the filament to that of the analytical solution, points were created at the center of the filament with 10 mm spacing. The temperature of the fluid at these points was then evaluated. These points are shown by Figure 12 - Figure 13.

Figure 12: Detail View of Points Created In Filament



Point186@jacket for filament-1@Duct and Filament Jacket Assembly
Point185@jacket for filament-1@Duct and Filament Jacket Assembly
Point184@jacket for filament-1@Duct and Filament Jacket Assembly
Point183@jacket for filament-1@Duct and Filament Jacket Assembly
Point182@jacket for filament-1@Duct and Filament Jacket Assembly

Global Coordinate System

■ All
☐ Density (Fluid)
☐ Density (Solid)
☐ Mass Fraction of Air
☐ Mass Fraction of Eastman 9921 PET 0.8 IV
☐ Pressure
☑ Temperature (Fluid)
☐ Temperature (Solid)
☐ Velocity

Figure 13: Point Parameters Selection in Flow Simulation

The analytical solution method's temperature distribution (updated to match the material properties and inlet flow temperature from the simulation) was then plotted against the simulation results, using the convection coefficient determined from the simulation. This is shown by Figure 14.

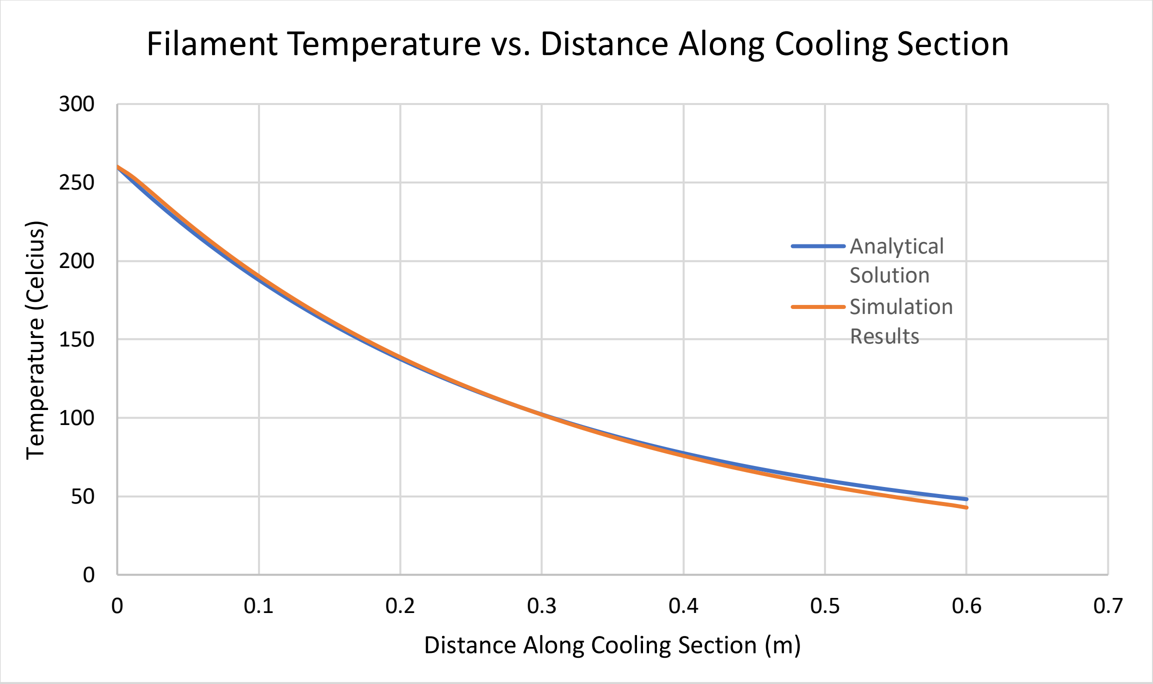


Figure 14: Filament Temperature vs Distance Along Cooling Section

By reviewing Figure 14, it is evident that the analytical solution method matches extremely well with that of the simulation, with an average percent error of just 2.8%.

Conclusion:

By reviewing Figure 14, it is evident that the simulation results match the analytical solution method very well. With the agreement between these results, it can be definitively determined that the fan selection for the cooling section is adequate with 5 fans at ~115 CFM/fan. Since each material will require different amounts of cooling, a variable fan speed “dimmer” will be wired into the fan circuit to allow the user to control the fan speed by a knob. This will allow for the system to be able to achieve well over the maximum fan speed, while being able to be adjusted for less thermally resistant materials and unpredictable situations.